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LETTER TO THE EDITOR

Observation of a complete band gap for liquid surface waves propagating over a periodically drilled bottom

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Abstract
We study both theoretically and experimentally the propagation of liquid surface waves over a periodically drilled bottom. A slab of a plate drilled with a triangular array of holes is placed in the middle of a liquid vessel. With the presence of point source waves generated on the one side of the plate, we can obtain propagative information of liquid surface waves through the observations of the outgoing waves on the other side of the plate. Our experiment confirms our previous theoretical predictions of the existence of complete band gaps for liquid surface waves propagating over a periodically drilled bottom.

Waves propagating in periodic structures are strongly modulated by the introduced periodicity [1]. As a result, multiple Bragg scatterings in periodic structures lead to complicated band structures. Between the bands there may exist band gaps within which wave propagation is absolutely forbidden. The most noticeable example is electronic band gaps in semiconductors. It has been found recently that band gaps can also exist in other kinds of waves propagating in periodic structures, for example, electromagnetic waves in photonic crystals [2–4] and sound waves in sonic crystals [5, 6]. The existence of band gaps renders a control of wave propagation in a desired way possible [4].

The propagation of a liquid surface wave over an uneven bottom is an important classical hydrodynamics problem [7, 8], which is of significance not only in basic research but also in potential applications such as coastal protection. In recent years, there has been considerable interest in the band structures of liquid surface waves propagating in periodic structures and the possibility of the existence of band gaps as well [9–19]. Complete band gaps (for all propagative directions) of liquid surface waves propagating in a periodic array of rigid cylinders were theoretically predicted [15, 17] and confirmed experimentally [19]. It has been found theoretically by the present authors that there could also exist complete band gaps for liquid surface waves propagating over a periodically drilled bottom [18]. However, there is no experimental confirmation so far. In the present work, we will present the experimental

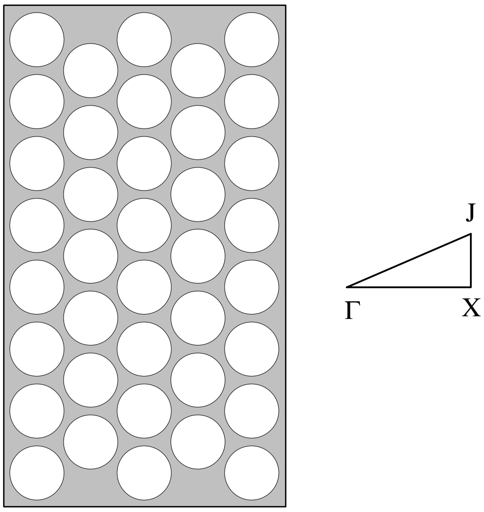

Figure 1. Schematic diagram (left panel) of the plate drilled with an array of holes arranged in a triangular lattice. The right panel shows the first irreducible Brillouin zone of the triangular lattice. Γ, X, and J are the centre, edge centre, and corner of the first Brillouin zone, respectively.

evidence in a visualized way in order to confirm our theoretical predictions. In fact, by using this direct visual method, many interesting phenomena such as Bloch waves over a periodically drilled bottom and domain walls over a drilled bottom with some disorder [20], Bloch-like waves over a quasiperiodically drilled bottom [21], and superlensing [22] and self-collimation [23] phenomena in liquid surface waves have been successfully observed.

The experimental set-up is basically similar to that used in the observation of superlensing [22] and self-collimation [23] phenomena in liquid surface waves. The experiment is carried out in a vessel, whose bottom is made of transparent glass. A slab of a flat methacrylate plate of thickness 4 mm is put on the bottom of the vessel. The methacrylate plate is drilled with holes arranged in a triangular lattice, shown schematically in figure 1. The holes are terminated horizontally in a direction perpendicular to the ΓX direction. Along the ΓX direction, there are nine columns of hole arrays. The lattice constant (inter-hole distance) is 15 mm and the radius of the holes is 6 mm. The plate is then covered with a special liquid, the same as used in [22] and [23]. The depth of the liquid is about 0.7 mm above the plate. Thus, the liquid depth over the drilled holes is 4.7 mm.

To determine whether there exists a complete band gap, plane-wave sources are commonly adopted. In principle, one should measure the transmittance spectra of a series of samples truncated in different directions. In the present work, a point source is used instead. The reason lies in the fact that the point source is a superposition of plane waves with wavevectors along all directions. In the experiment, the point source generator is placed on one side of the plate, about 4 mm apart. The driven amplitude of the point source generator is kept rather small in order to avoid nonlinear effects. A halogen lamp is hung about 1 m above the vessel. Projected images of liquid surface wave patterns can be visualized or obtained by a digital camera on the screen with the help of the mirror placed below the vessel. A detailed description of the experimental set-up can be found elsewhere [22].

To obtain insight into the propagation properties, the band structures for liquid surface waves propagating over a bottom drilled with an array of a triangular lattice are calculated, as shown in figure 2. The propagation of liquid surface waves is dealt with by using a mild-slope

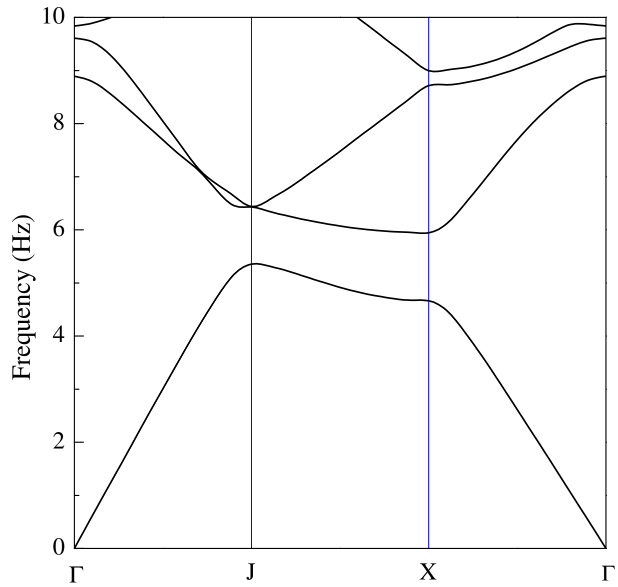

Figure 2. Calculated band structures for liquid surface waves propagating over a bottom drilled with an infinite array of holes arranged in a triangular lattice. The parameters used in the calculations are the same as used in the experiment.

(This figure is in colour only in the electronic version)

equation [24, 25], which can give a successful description of the evolutions of liquid surface waves over a bottom with varying topography. The mild-slope equation is solved based on a plane-wave expansion method developed by us, described in detail elsewhere [18]. The following dispersion relation of liquid surface waves [14] is used in our calculations:

$$\omega^2 = gk(1 + Tk^2/\rho g)\tanh(kh), \qquad (1)$$

where k is the local wavenumber, h is the liquid depth, T is the liquid surface tension, g is the gravitational acceleration, and ρ is the liquid density. For the periodically drilled bottom, the liquid depth h is a spatially varied function. Within the framework of the plane-wave expansion method, the problem of solving the mild-slope equation, by imposing the Bloch theorem, becomes an eigenvalue problem [18]. In our calculations, 285 plane waves are used; this gave a satisfactory convergence.

For frequencies ranging from 0 to 4.66 Hz (the frequency of the first band at X), waves can propagate along all directions. However, a complete band gap exists between the first and second bands. Its lower edge is defined by the highest frequency (5.35 Hz) of the first band at J, while the upper edge is defined by the lowest frequency (5.94 Hz) of the second band at X. For waves with frequencies located into this complete band gap, no waves can propagate. It is interesting that partial band gaps along certain directions also exist. For example, along the ΓX direction, there is a partial band gap with frequency ranging from 4.66 to 5.94 Hz, defined by the highest and lowest frequencies of the first and second bands at X. For frequencies within this partial band gap, waves cannot propagate along the ΓX direction, but they may propagate along other directions. The frequency range from 5.35 to 6.44 Hz, defined by the highest and lowest frequencies of the first and second bands at J, corresponds to a partial band gap along the ΓJ direction. The existence of partial band gaps can influence wave propagation considerably. Waves with frequencies from 4.66 to 5.35 Hz cannot propagate along the ΓX direction, but

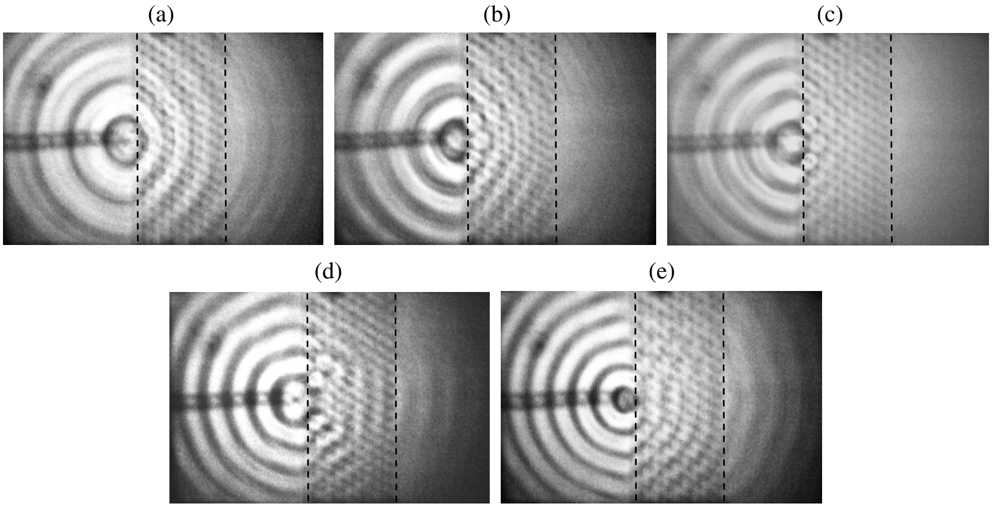

Figure 3. Snapshots of liquid surface waves at different driven frequencies (a) 3.87, (b) 5.0, (c) 5.6, (d) 6.24, and (e) 6.75 Hz. The plate (between two horizontal lines) drilled with a triangular array of holes is placed in the middle of the vessel. The point source generator is located on the left-hand side of the plate, about 4 mm apart. The straight grey image that extends from the left-hand side to the right-hand side of the plate is the shadow of a tube of the point source generator.

they can propagate along the ΓX direction, since this frequency range corresponds to the first band. Similarly, waves with frequencies from 5.94 to 6.44 Hz can propagate along the ΓX direction, but they cannot propagate along the ΓJ direction.

As is known, a point source can be decomposed as a superposition of all possible Bloch waves with well defined wavevectors. In the presence of a point source wave generated on one side of the drilled plate, the propagation scenario is as follows. The point source wave first enters the hole arrays across the interface via coupling to the Bloch waves of the periodic structure, propagates in the hole array region in the form of Bloch waves, and finally leaves the hole array region. The outgoing wave is strongly modulated by the periodical structure in the hole array region. Information on wave propagation in the periodic structure can be obtained through observing the outgoing waves on the right-hand side of the drilled plate. For frequencies located into a complete band gap, no wave propagation is allowed. Consequently, no transmitted wave can be observed. In contrast, if transmitted waves are observed, the frequencies of these waves should be located into certain bands.

Figure 3 shows snapshot patterns of the liquid surface waves at different driven frequencies in the presence of a point source placed on the left-hand side of the plate drilled with a triangular array of holes. Circle-like stripes on the left-hand side of the plate correspond to the point source waves driven by the point source generator. The propagation of liquid surface waves inside the triangular array of holes and those transmitted across the plate can be clearly visualized.

In the presence of a point source wave generated on one side of the periodically drilled holes, the scenario of its propagation is as follows. The point source wave first enters the plate across the interface via coupling to the Bloch waves of the periodic structure, traverses the plate in the form of Bloch waves, and finally leaves the plate. The outgoing waves from the other side of the plate are strongly modulated by the plate. It can be seen from figure 3 that at 3.87 Hz (figure 3(a)) the transmitted wave pattern is circle-like. This implies that waves can propagate along all directions in the triangular array of holes. At 5.0 Hz (figure 3(b)), there is almost no propagation along the ΓX direction, but there are propagating waves along the ΓJ direction. This can be understood by the fact that this frequency is located in a partial band gap along the ΓX direction, hindering the propagation along this direction. However, propagation along the ΓJ direction is allowed since this frequency is located into the first band along the ΓJ direction. At 5.6 Hz (figure 3(c)), no wave pattern on the right-hand side of the plate is observable, indicating that this frequency is just located in the complete band gap. At 6.24 Hz (figure 3(d)), propagation along the ΓX direction is observed, while there is no propagation along the ΓJ direction. This propagative feature is due to the existence of a partial band gap along the ΓJ direction. Wave propagation that dominates along one direction can lead to the self-collimation effect, found in photonic crystals [26, 27], sonic crystals [28], and in liquid surface waves propagating in periodic structures [23] as well. At 6.75 Hz (figure 3(e)), it is observed again that propagation can occur along all directions. The observed frequency range of the complete band gap runs approximately from 5.4 to 5.8 Hz, supporting our theoretical prediction.

In our experimental set-up, there are two interfaces between the regions with and without drilled hole arrays. For an interface formed by two different media, wave reflection and refraction are expected. On the right-hand side of the region with drilled hole arrays, there are only transmitted waves. For frequencies within the gap region, no transmitted waves exist. On the left-hand side of the region with drilled hole arrays, however, the wave patterns are the superposition of the point source waves and the reflected waves. From figure 3, we can see that the wave patterns on the left-hand side show some distortions from those of the point source waves. Moreover, there are also some variations in brightness for the same circle-like stripe. These features can be understood by the interference of the reflected waves and the point source waves.

In summary, through observations of the evolution of point source waves traversing the slab of a plate drilled with a triangular array of holes, we have clearly demonstrated the existence of a complete band gap for liquid surface waves propagating over a periodically drilled bottom, confirming our previous theoretical predictions. In addition, the influence of the partial band gaps is also visualized. The existence of band gaps in liquid surface waves could lead to many novel phenomena. As with photonic crystals, we can take advantage of band gaps to manipulate liquid surface wave propagation, which may result in important applications such as in coastal protection or in energy generation. In addition, liquid surface wave experiments are an excellent tool for exploring novel wave phenomena in classical waves in a lively visualized way.

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